UNITED STATES
DEPARTMENT OF THE INTERIOR
BUREAU OF RECLAMATION

USE OF AN ELECTRONIC COMPUTER TO OBTAIN FLOW NETS FOR A CHANNEL WITH 90° INTO-THE-FLOW OFFSETS

Hydraulics Branch Report No. Hyd. 500

DIVISION OF RESEARCH

OFFICE OF CHIEF ENGINEER
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Report No. Hyd-500
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Subject: Use of an electronic computer to obtain flow nets for a channel with 90° into-the-flow offsets

PURPOSE

This study was made to develop a computer program for evaluating an exact mathematical expression for flow in a channel with various 90° into-the-flow offsets. The program would be used to compute the pressure coefficient, the vertical and horizontal components of the relative velocity, and the resultant relative velocity at any point in the channel.

CONCLUSIONS

1. An electronic computer can be used effectively to evaluate an exact mathematical relationship for flow in a channel having 90° into-the-flow offsets. The graphical representation of this evaluation yields the desired flow net.

2. The computer program can be used to obtain the pressure coefficient, the vertical and horizontal components of the relative velocity, and the resultant relative velocity at all points in the flow.

3. The flow net can be used to determine the proper placement of measuring stations that are essentially unaffected by the offsets and to qualitatively obtain the distribution of pressure and velocity around the offset.

INTRODUCTION

Diagrams of streamlines and velocity potentials, commonly called flow nets, are used to obtain greater insight into the dynamics of fluid problems. For instance, flow nets have been used quite successfully to obtain the distribution of pressure on streamlined struts, as well as predicting the discharge coefficient for flows over weirs or under gates. Flow nets have also been used to determine the distance that a measuring point must be placed upstream from a pier or other flow disturbing element to yield results that are essentially free from the effects of the disturbing elements.

The determination of a flow net is generally accomplished by means of mathematical evaluations of the complex potential of flow, graphical and relaxation techniques, electrical analogs, numerical integrations, or by Monte Carlo methods.[1/][2/] Exact mathematical solutions can be derived for certain simple configurations, but the exact solution is generally too complex to evaluate in a reasonable length of time by manual methods. Due to the laborious procedures that are generally required to obtain flow nets, the nets are frequently not fully utilized in the dynamic analysis of fluid problems. However, the laborious and time-consuming part of these methods can be eliminated through the use of electronic computers. A computer can obtain flow nets using any of the previously mentioned methods with the exception of the graphical method and the electrical analog.

An example of the successfuly application of electronic computers to evaluate the mathematical expression for a flow net is found in a recent study concerning cavitation. This research study was conducted by the Hydraulics Branch, Bureau of Reclamation, and consisted of raising various offset shapes into the flow through a rectangular conduit. The upstream pressure and the velocity required to cause cavitation to form at the offset were observed and recorded.

The piezometer tap used to determine the upstream pressure was by necessity located only a short distance upstream from the test section. It was not known if this tap was far enough upstream to be essentially free from the effect of the various offsets. Therefore, a flow net solution was obtained for the case of a 90° offset. The 90° shape was selected because it produces the most pronounced effect on the upstream measuring station. The flow net obtained indicated the magnitude of error that could be expected in the measurement with various ratios of offset heights to passage heights. A good correlation with the flow net solution was then obtained through an experimental verification. The experimental and analytical studies indicated that a flow net could be used to estimate pressures and velocities in the vicinity of the offset.

[1/] Rouse, H., "Engineering Hydraulics," John Wiley and Sons, Inc., New York, pp 15-22, 1950

[2/] McNown, J. S., Hsu, E. Yih, C., "Application of the Relaxation Technique in Fluid Mechanics," Transactions ASCE, Volume 120, pp 650-686, 1955

This report is a presentation of the above example and includes the general mathematical expression, the computer program, solutions for two distinct offset ratios, and the experimental verification.

INVESTIGATION

The investigation consisted of two parts. The first dealt with the theoretical case of flow with an ideal fluid and included considerations concerning the derivation of the general mathematical expression, methods used to compute the pressure coefficient and the relative velocity at any point, a computer program to evaluate the mathematical expression, and examples of flow nets for two discrete offset ratios. The second part compared the theoretical results with those obtained experimentally, using pressure coefficients as the basis for comparison.

General Mathematical Expression

With an ideal fluid, the flow is assumed to be incompressible and nonviscous. For purposes of deriving an expression for flow past an offset, the flow is also assumed to be steady state and two-dimensional. That is (1) the quantity of flow past a point neither increases nor decreases with time, and (2) the flow is considered to lie in a plane with no secondary flow entering or leaving that plane.

For purposes of clarity, the following definitions of standard terms concerning flow nets are summarized as follows:

$V = V_1 + V_2 i$	is a complex number denoting the velocity vector of a particle of fluid at any point (x,y). The x and y components of velocity are $V_1(x,y)$ and $V_2(x,y)$, respectively.
$\varphi(x,y)$	is the VELOCITY POTENTIAL, and curves of $\varphi(x,y) = c$ are EQUIPOTENTIAL LINES. The function $\varphi(x,y)$ is called the "velocity potential" because of the relations: $V_1 = \partial\varphi/\partial x$ and $V_2 = \partial\varphi/\partial y$

$\psi(x,y)$ is the STREAM FUNCTION, and curves of $\psi(x,y) = c$ are STREAMLINES.

$F(z) = \varphi(x,y) + \psi(x,y)$ is the COMPLEX POTENTIAL OF FLOW in the z plane (real plane).

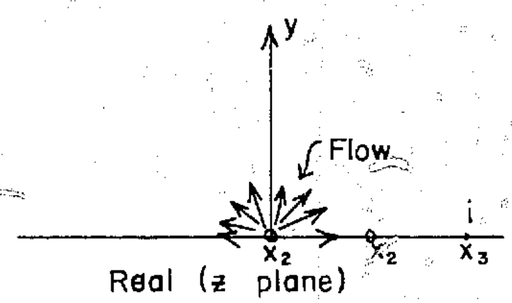
$F(w) = \varphi(u,v) + \psi(u,v)$ is the COMPLEX POTENTIAL OF FLOW in the w plane (complex plane).

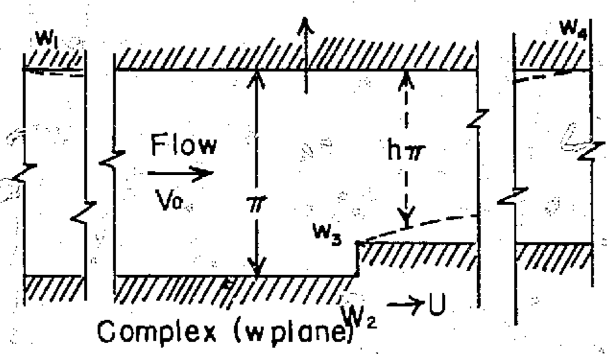
A FLOW NET is the graphical representation of the complex potential of flow.

The subscript o refers to points in the zone of uniform flow; no subscript denotes points in the region of altered flow.

A detailed derivation of the general expression for the complex potential of flow past a 90° offset can be found in most standard books on Theoretical Hydrodynamics.[3/4/] Therefore, only the most important parts of the derivation are included in this report.

The derivation is based on the Schwarz-Christoffel Transformation which can be used to transform a source at the origin of a real plane into flow past an offset in a complex plane. The following sketch illustrates the transformation in which the x-axis in the real plane is transformed into a four-sided polygon in the imaginary or complex plane.





3/Churchill, "Complex Variables and Applications," McGraw Hill, 2nd Edition, 1960

4/Milne, L. M., Thompson, "Theoretical Hydrodynamics," MacMillan Company, New York, 1950

Through the use of the transformation and knowing the complex potential of flow for a source, the following two equations are derived:

$$u + vi = h \ln\left(\frac{1+s}{1-s}\right) - \ln\left(\frac{h+s}{h-s}\right) \tag{1}$$

$$s^2 = \frac{e^{\varphi/v_o}(\cos \varphi/v_o + i \sin \psi/v_o) - h^2}{e^{\varphi/v_o}(\cos \psi/v_o + i \sin \psi/v_o) - 1} \tag{2}$$

These equations give the implicit relationship between the complex potential of flow past an offset and coordinates in the w-plane. In other words, given values for φ/v_o and ψ/v_o, a value of s can be computed, where s is a complex number. The value of s when substituted into Equation (1) gives the values of U and V that correspond to the assumed values of φ/v_o and ψ/v_o. These are essentially the steps performed by the computer in evaluating Equations (1) and (2).

Three important observations concerning the flow, which arise through the detailed derivations, are: (1) at w_3 the velocity becomes infinitely large positively, (2) at w_2 the velocity is zero making w_2 a stagnation point, and (3) the total pressure on the vertical face of the offset is infinitely large negatively. Observations (1) and (3) clearly indicate that the flow net cannot be used to obtain meaningful results on the offset face nor in the immediate vicinity of the offset corner because real fluids are not physically capable of producing these results. However, at all other points in the flow net, conditions are compatible with possibilities that might occur with the flow of real fluids.

Relative Velocity and Pressure Coefficient

Two methods are given to compute the relative velocity and the pressure coefficient. The first method is approximate and usually is used when the flow net is known and the mathematical solution is unknown. The second method is exact, but restricted to flow past a 90° offset. However, similar expressions can be derived for other cases in which the mathematical expression is known.

<u>Approximate Method</u>. Let Δs be the incremental distance between equipotential lines, and

Δn be the incremental distance between streamlines.

Through the detailed derivation it was found that

$$\frac{\partial\varphi}{\partial x} = \frac{\partial\psi}{\partial y} \quad \text{or} \quad \frac{\Delta\varphi}{\Delta s} = \frac{\Delta\psi}{\Delta n}$$

Therefore, since the velocity is a function of φ (from the definition of terms):

$$\frac{V_a}{V_b} = \frac{(\Delta\varphi/\Delta s)_a}{(\Delta\varphi/\Delta s)_b} = \frac{(\Delta\psi/\Delta n)_a}{(\Delta\psi/\Delta n)_b}$$

Taking equal increments of $\Delta\varphi$ and $\Delta\psi$ (making the flow net "square") and in addition letting the point "a" represent a point in the zone of altered flow and the point "b" represent a reference point in the zone of uniform flow, yields:

$$\frac{V}{V_o} = \frac{\Delta s_o}{\Delta s} = \frac{\Delta n_o}{\Delta n} \tag{3}$$

The pressure coefficient can be obtained by writing Bernoulli's equation between two points of equal elevation and assuming no loss of energy between the two points. This yields:

$$\rho\,\frac{V_a^2}{2} + P_a = \rho\,\frac{V_b^2}{2} + P_b$$

or

$$P_a - P_b = \rho\,\frac{V_b^2}{2} - \rho\,\frac{V_a^2}{2}$$

and

$$\frac{P_a - P_b}{\rho\, V_b^2/2} = 1 - \left(\frac{V_a}{V_b}\right)$$

Again, with "a" and "b" as defined above, the pressure coefficient becomes:

$$C_p = \frac{P - P_o}{\rho V_o^2/2} = 1 - \left(\frac{V}{V_o}\right)^2 = 1 - \left(\frac{\Delta s_o}{\Delta s}\right)^2 = 1 - \left(\frac{\Delta n_o}{\Delta n}\right)^2 \tag{4}$$

Thus, Equations (3) and (4) give the approximate values of velocity and pressure coefficient at any point with reference to a fixed point. The incremental units are taken from measurements on a flow net. A knowledge of the mathematical relations of Equations (1) and (2) is not needed to solve for Equations (3) and (4).

Exact Method. The following equation is derived as a part of the detailed derivation:

$$V_1 - V_2 i = \frac{V_o}{h}\left(\frac{z - h^2}{z - 1}\right)^{1/2} = \frac{V_o s}{h} \tag{5}$$

or

$$\frac{V_1}{V_o} - \frac{V_2}{V_o i} = \frac{s}{h}$$

This equation yields the vertical and horizontal components of the velocity at any point with respect to the velocity at the reference point, a result which can only be crudely approximated in the previous method.

The resultant relative velocity at any point can be determined from the following equation:

$$\frac{V}{V_o} = \frac{|V_1 - V_2 i|}{V_o} = \frac{|s|}{h} = \frac{\sqrt{(V_1)^2 + (V_2)^2}}{h} \tag{6}$$

where the absolute value symbol, $|\ |$, means that the square root of the sum of the squares of the real and imaginary parts is taken.

The expression for the pressure coefficient at any point can be derived from Equations (4) and (6). This expression is:

$$C_p = \frac{P - P_o}{\rho V_o^2/2} = 1 - \frac{s^2}{h^2} \tag{7}$$

An examination of Equations (5), (6), and (7) reveals that the reference point is defined mathematically as a point in the region of uniform flow. For the approximate method, the reference point is selected from a flow net in a region where the flow is essentially uniform. Although this point can usually be selected to give any desired degree of accuracy, the fact that a reference point must be chosen makes the approximate method undesirable for programing with an electronic computer.

The Computer Program

As mentioned previously, the basic computer program consists of computing U and V coordinates from given values of φ/V_o and ψ/V_o. The relationship between these quantities is given in Equations (1) and (2). An extension of the program allows the computation of the vertical and horizontal components of the relative velocity, the resultant relative velocity, and the pressure coefficient at the point for the assumed values of φ/V_o and ψ/V_o. These relations yield information concerning the nature of the flow in the vicinity of the offset and also permit a comparison of the theoretical flow net with experimental data.

Both positive and negative values are obtained when taking the square root of Equation (2). If both of these values were substituted into Equation (1), the result would be the flow net for a rectangular plate located in the center of the conduit. The computer program eliminates this problem by using only the positive root.

The program, which was written using FORTRAN, results in two final parts.[5] These parts are embodied in an Object Deck, which contains the necessary commands to tell the computer what to do, and a Data Deck, which contains the offset ratio and discrete values of φ/V_o and ψ/V_o.[6] The Object Deck is applicable to any offset ratio or any number of φ/V_o and ψ/V_o combinations.

[5] See Appendix, FORTRAN

[6] See Appendix, Mathematical Steps for Flow Net Computation, Block Diagrams, FORTRAN Coding Form (Source Program)

The Data Deck contains all the variations for various problems and must have the following format:

Enter the offset ratio, h, in a 14-digit floating point field

Enter the number of data points, n (φ/V_o and ψ/V_o combinations) in a 5-digit fixed point field

Enter the data points, φ/V_o first and ψ/V_o second, each in a 14-digit floating point field.

The Data Deck can be made from two parts, one part containing only the offset ratio, the other part containing the number of data points and the φ/V_o and ψ/V_o combinations. In this manner the same data points can be used with many different offset ratios. Only the new offset ratio would then need to be punched for different individual solutions. The first part of the Data Deck is inserted into the machine as indicated in the Appendix under FORTRAN. As soon as the computer has read the offset ratio the additional data are read.

The following restrictions and considerations apply to the selection of the possible φ/V_o and ψ/V_o combinations for use as data:

1. ψ/V_o varies between zero and π. Therefore, π must be divided by an integer in order to divide the channel depth into a whole number of equal increments.

2. A "square" flow net will result from choosing equal increments of φ/V_o and ψ/V_o.

3. For the case of ψ/V_o = zero, the following observations concerning φ/V_o can be made:

(a) If φ/V_o is less than $\ln h^2$, V is equal to zero and values of U will be computed.

(b) If φ/V_o is equal to $\ln h^2$, U and V are both equal to zero.

(c) If φ/V_o is greater than $\ln h^2$, but less than zero, U is equal to zero and V is greater than zero, but less than $(1.00 - h)$. Care must be taken in the selection of φ/V_o. As φ/V_o approaches zero, the computer must work with very large numbers. In the event the capacity of the computer is exceeded, erroneous values of V will result.

(d) If φ/V_o is equal to zero, then U is equal to zero and V is equal to $(1.00 - h)$. The pressure coefficients and velocity components are infintely large. The computer will print $\varphi/V_o = 0$, $\psi/V_o = 0$, $U = 0$, $V = 0$, for this case.

(e) If φ/V_o is greater than zero, then V is equal to $(1.00 - h)$, and values of U can be computed. Care must also be exercised in choosing values of φ/V_o near zero for the same reason as given in (3).

The computer program is written so that every division is tested to insure that the divisor is not equal to zero. In addition, the program tests to insure that every logarithm is defined. If the divisor is equal to zero or if the logarithm of zero must be taken, the computer will print out a coded number. This number indicates the specific reason for the print out.[7] The computer will then select the next two data points and continue the operation.

Example

Flow nets were computed for the offset ratios, $h = 0.833$, and $h = 0.917$ (Figure 1). These ratios represent a 1/2-inch offset and a 1/4-inch offset into a 3-inch deep stream, respectively. The influence of the various offsets can be seen through the curvature of the $\varphi/V_o = c$ lines. The greater the curvature, the greater the influence. These nets clearly indicate, without further computations, that a piezometer which was one conduit depth upstream $(\varphi/V_o \approx -\pi)$ from either of the two offsets would be in a region that is essentially unaffected by the offset.

Comparison of Theoretical Results with Experimental Results

In order to establish whether or not a flow net could be used to obtain a quantitative as well as qualitative result, tests were conducted in the laboratory with flow past a 90° offset. An offset ratio of $h = 0.833$ was chosen for the comparison of theoretical values with those found experimentally. The pressure coefficients were used as the basis of comparison (Figure 2A).

The main sources of discrepancy that might be expected between the experimental and theoretical results would be (1) assumptions made in the original derivation, (2) assumptions made in computing the pressure coefficient, (3) piezometers not giving the true static pressure for conditions of incipient cavitation.[8]

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[7] See Appendix Coded Number Interpretation

[8] Daily, J. W., Lin, J. D., Broughton, R. S., "Turbulence and Static Pressure in Relation to Inception of Cavitation," IAHR 9th Convention. "* * * The measured pressures in the main flow beyond the boundary layer are lower than at the wall. * * *," 1961

An examination of the magnitudes of the discrepancies revealed that the energy loss due to the sudden contraction would result in the largest discrepancy between the two methods (excluding pressure coefficients on the vertical face of the offset and the channel floor immediately downstream from the offset). Experiments by Weisbach indicate that the magnitude of the losses due to the contraction is approximately 0.045 $(v^2/2g)$.[9] However, tests in the cavitation study indicate a value of 0.080 $(v^2/2g)$ (Figure 2B). The difference between the theoretically determined pressure coefficient and that obtained through experiment can be obtained from the experimental loss coefficient curve (Figure 2A). The magnitude of this difference is given by $\Delta C_p = K/h^2$.[10] It should be noted that this is the maximum discrepancy that is to be expected for pressure coefficients on the channel roof.

Immediately upstream from the face of the offset a vortex was observed. This vortex indicates that separation of the flow from the channel floor exists upstream from the offset. This separation and its accompanying loss of energy are the main reason for the difference between the theoretical curve and the experimental curve in the region upstream from the offset (Figure 2A). For the ratio, h = 0.833, the magnitude of this loss becomes negligible upstream from the offset at a distance of 0.3 times the conduit depth.

A spot check of theoretical pressure coefficients as compared to the experimentally determined values indicated that qualitative results could be obtained in the vicinity of the offset. However, due to the fact that separation also occurs at the corner of the offset, a good correlation is not possible for points on the conduit floor immediately downstream from the offset.

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[9] Rouse, H., Ibid, p 414, the "* * * data is based upon contraction coefficients given by Weisbach for free rather than submerged efflux; the resulting values must hence be regarded as rough approximations."

[10] See Appendix, Derivation of Pressure Coefficient Discrepancy

APPENDIX

FORTRAN[11]

An electronic computer is a very useful and high-speed tool, but one that is capable of performing only relatively simple steps and must be told when to perform each step. The process of separating a problem into the many simple steps that the computer can perform is called "programing."

Several developments are available which considerably shorten and simplify the programing process. In one of these developments, called FORTRAN (FORmula TRANslation), the computer assists in writing the program. With FORTRAN, the engineer needs only to express his problem in a series of statements or formulas. The computer translates these into a language that the computer can understand through the use of several secondary programs or "compilers." The engineer's statements consist of arithmetic operations, calls for input or output, commands to designate the sequence in which the statements are to be performed, and statements which provide information but cause no action. The combination of this series of statements is called the FORTRAN program. This program and the compilers are the only tools needed to successfully program a problem with the FORTRAN system.

In general, a program is solved along the following lines.[12] The FORTRAN program is punched onto a paper tape or cards by means of a flexowriter or card punching machine. The resulting tape or card deck is called the source program tape or card deck. Before utilizing the source program, another program called the FORTRAN compiler is read into the computer. Under control of the FORTRAN compiler program, the computer reads the source program and translates it into machine instructions and punches these instructions on cards or tape. The program which the machine punches is known as the object program. The object program with any necessary data is then read into the computer and the program is executed and results are typed out or punched on paper tape or cards. In the case of punched output, the tape or cards may be inserted into equipment which will translate the punches into typed output.

Mathematical Steps for Flow Net Computation

The two basic equations which are evaluated in order to obtain the flow net are:

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[11] McCracken, D. D., "A Guide to Fortran Programing," John Wiley and Sons, Inc., New York, 1961

[12] Punched paper tape and punched cards are two common methods that are used for the input and output of data from an electronic digital computer.

$$u + iv = w = h \ln(1+s) - h \ln(1-s) - \ln(h+s) + \ln(h-s) \tag{1}$$

and

$$s^2 = \frac{(e^{\varphi/V_o} \cos \psi/V_o - h^2) + i(e^{\varphi/V_o} \sin \psi/V_o)}{(e^{\varphi/V_o} \cos \psi/V_o - 1) + i(e^{\varphi/V_o} \sin \psi/V_o)} \tag{2}$$

These equations were broken down into the following simpler steps for programing with an electronic computer:

$$e^{\varphi/V_o} \cos \psi/V_o - h^2 = a \tag{3a}$$

$$e^{\varphi/V_o} \sin \psi/V_o = b \tag{3b}$$

$$e^{\varphi/V_o} \cos \psi/V_o - 1 = a + h^2 - 1 = c \tag{3c}$$

Then Equation (2) becomes

$$s^2 = \frac{a+ib}{c+ib} = \frac{ac+b^2}{b^2+c^2} + i\,\frac{bc-ab}{b^2+c^2} = \frac{xx}{b^2+c^2} + i\,\frac{yy}{b^2+c^2} \tag{4}$$

A complex number may also be written in the form

$$s^2 = r(\cos\theta + i\sin\theta)$$

Taking the square root gives,

$$s = \sqrt{r}(\cos\theta/2 + i\sin\theta/2) \tag{5}$$

where

$$r = \frac{\sqrt{xx^2 + yy^2}}{b^2 + c^2} \quad \text{and} \quad \theta = \tan^{-1}\frac{yy}{xx} \quad \text{for} \quad -\pi \le \theta < \pi$$

Equation (1) may now be evaluated by using Equation (5) in the following manner:

Let

$$1 + \sqrt{r}\cos\theta/2 = d; \quad 1 - \sqrt{r}\cos\theta/2 = f; \quad h + \sqrt{r}\cos\theta/2 = g$$

$$- \sqrt{r}\cos\theta/2 = p; \quad \sqrt{r}\sin\theta/2 = si$$

Then

$$h\ln(1+s) = h\ln(\sqrt{d^2+(si)^2}) + i(h\tan^{-1} si/d);$$

$$h\ln(1-s) = h\ln(\sqrt{f^2+(si)^2}) - i(h\tan^{-1} si/f);$$

$$\ln(h+s) = \ln(\sqrt{g^2+(si)^2}) + i(\tan^{-1} si/g);$$

$$\ln(h-s) = \ln(\sqrt{p^2+(si)^2}) - i(\tan^{-1} si/p)$$

Thus

$$U = [h\ln(\sqrt{d^2+(si)^2}) - h\ln(\sqrt{f^2+(si)^2})$$

$$- \ln(\sqrt{g^2+(si)^2}) + \ln(\sqrt{p^2+(si)^2})]/\pi$$

and

$$V = +[h\tan^{-1} si/d + h\tan^{-1} si/f - \tan^{-1} si/g$$

$$- \tan^{-1} si/p]/\pi$$

The vertical and horizontal components of velocity are computed as follows:

$$V_1/V_o = \text{horizontal component} \qquad V_2/V_o = \text{vertical component} \tag{6}$$

and $V_1/V_o = V_2/V_o \cdot i = s$

Thus

$$\frac{V_1}{V_o} = \frac{\sqrt{r}\ \cos\ \theta/2}{h} = \frac{sr}{h}$$

and

$$\frac{V_2}{V_o} = - \frac{\sqrt{r}\ \sin\ \theta/2}{h} = - \frac{si}{h}$$

The resultant velocity is simply the vector sum of the two velocity components.

Let V_r/V_o = resultant velocity

Then $V_r/V_o = \sqrt{(V_1/V_o)^2 + (V_2/V_o)^2} = \sqrt{r}/h$

The pressure coefficient is computed from the relationship

$$C_p = 1 - \frac{|s^2|}{h^2} \tag{7}$$

where

$$|s^2| = r$$

Then

$$C_p = 1 - r/h^2$$

PROGRAM FOR
FLOW NET FOR 90 DEGREE OFFSETS


```
   1 FORMAT (I5)
   3 FORMAT (8F14.6)
 1040FORMAT (112H                PHI              PSI             U
    1 V             VVEL            HVEL            RVEL             CP)
 108 FORMAT ( 1H1, 5X, 3H H=, F10.6)
C      READ DATA
 107 READ INPUT TAPE 5, 3, H
     WRITE OUTPUT TAPE 6, 108, H
     READ INPUT TAPE 5, 1, N
     WRITE OUTPUT TAPE 6, 104
     DO 45 J = 1, N
     READ INPUT TAPE 5, 3, PHI, PSI
C      TEST PHI AND PSI
     EXPI = EXPF (PHI)
     HSQ = H*H
     TEST = EXPI - HSQ
     HTEST = EXPI -1.0
     IF (PSI) 21, 17, 21
  17 IF (TEST) 21, 18, 19
  18 U = 0.0
     V = 0.0
     VVEL = 0.0
     HVEL = 0.0
     RVEL = 0.0
     CP = 1.0
     GO TO 42
  19 IF (HTEST) 21, 20, 21
  20 U = 0.0
     V = 1.0 - H
     GO TO 44
C      COMPUTATION OF S REAL AND S IMAGINARY
  21 A = EXPI * COSF (PSI) - HSQ
     B = EXPI * SINF (PSI)
     C = A + HSQ - 1.0
     XX = A*C + B*B
     YY = B*(C-A)
     R = SQRTF(XX*XX + YY*YY) / (B*B + C*C)
     RR = SQRTF (R)
     CALL ANGLE (XX, YY, THETA)
     HTHETA = THETA / 2.0
     SR = RR * COSF (HTHETA)
     SI = RR * SINF (HTHETA)
C      COMPUTATION OF U AND V COORDINATES
     D = 1.0 + SR
     F = 1.0 - SR
     G = H + SR
     P = H - SR
  22 IF (D) 24, 23, 24
  23 WRITE OUTPUT TAPE 6, 100
 100 FORMAT (30H THE VALUE 1.0 + S EQUALS ZERO)
     GO TO 43
```

```
   24 IF (F) 26, 25, 26
   25 WRITE OUTPUT TAPE 6, 101
  101 FORMAT (28H THE VALUE 1.0-S EQUALS ZERO)
      GO TO 43
   26 IF (G) 28, 27, 28
   27 WRITE OUTPUT TAPE 6, 102
  102 FORMAT (28H THE VALUE H + S EQUALS ZERO)
      GO TO 43
   28 IF (P) 30, 29, 30
   29 WRITE OUTPUT TAPE 6, 103
  103 FORMAT (26H THE VALUE H-S EQUALS ZERO)
      GO TO 43
   30 IF (YY) 39, 31, 39
   31 IF (PSI) 34, 32, 34
   32 IF (TEST) 35, 18, 33
   33 IF (HTEST) 36, 20, 37
   34 V = 1.0
      GO TO 38
   35 V = 0.0
      GO TO 38
   36 U = 0.0
      GO TO 40
   37 V = 1.0 - H
   380U =(H*LOGF(ABSF(D)) - H*LOGF(ABSF(F)) - LOGF(ABSF(G)) + LOGF(ABSF
     1 (P))) / 3.1415926
      GO TO 41
   39 U1 = H*LOGF(SQRTF(D*D + SI*SI))
      U2 = H*LOGF(SQRTF(F*F + SI*SI))
      U3 = LOGF(SQRTF(G*G + SI*SI))
      U4 = LOGF(SQRTF(P*P + SI*SI))
      U = (U1 - U2 - U3 + U4) / 3.1415926
   40 CALL ANGLE (D, SI, ALPHA)
      CALL ANGLE (F, SI, BETA)
      CALL ANGLE (G, SI, GAMMA)
      CALL ANGLE (P, SI, DELTA)
      V = (H*(ALPHA + BETA) - GAMMA - DELTA) / 3.1415926
C     COMPUTATION OF VERTICAL AND HORIZONTAL VELOCITY COMPONENTS
   41 HVEL = SR/H
      VVEL = (-SI/H)
C     COMPUTATION OF THE RESULTANT VELOCITY
      RVEL = RR / H
C     COMPUTATION OF THE PRESSURE COEFFICIENT
      CP = 1.0 - R / HSQ
   42 WRITE OUTPUT TAPE 6,  3,  PHI, PSI, U, V, VVEL, HVEL, RVEL, CP
      GO TO 45
   43 WRITE OUTPUT TAPE 6, 105, PHI, PSI
  105 FORMAT ( 5X,  5H PHI=, F14.6, 10X, 5H PSI=, F14.6)
      GO TO 45
   44 WRITE OUTPUT TAPE 6, 106, PHI, PSI, U, V
  1060FORMAT (5X, 5H PHI=, F14.6, 5X, 5H PSI=, F14.6, 5X, 3H U=, F14.6,
     1 5X, 3H V=, F14.6)
   45 CONTINUE
```

```
GO TO 107                                              124
END(1,0,0,0,0,0,1,0,0,0,0,0,0,0,0)
```

PLACEMENT OF ANGLES IN PROPER QUADRANT 7/02/63 PAGE 1

```
      SUBROUTINE ANGLE ( X, Y, RHO )
      IF (X) 6, 10, 14
    6 IF (Y) 7, 8, 9
    7 RHO = ATANF (Y/X) - 3.1415926
      RETURN
    8 RHO = (-3.1415926)
      RETURN
    9 RHO = ATANF (Y/X) + 3.1415926
      RETURN
   10 IF (Y) 11, 12, 13
   11 RHO = (-1.5707963)
      RETURN
   12 RHO = 0.0
      RETURN
   13 RHO = 1.5707963
      RETURN
   14 IF (Y) 15, 12, 15
   15 RHO = ATANF (Y/X)
      RETURN
      END(1,0,0,0,0,0,0,0,0,0,0,0,0,0,0)
```

Coded Number Interpretation

These pause numbers are used to let the operator know the reason a certain value cannot be computed and to allow flexibility in data insertion. The pause numbers have the following meanings;

Pause Number	Meaning
1	Used in SUBRØUTINE ANGLE. In the expression rho = $\tan^{-1} y/x$, both x and y are equal to zero. For this case the angle was defined as equal to zero.
2	Used in MAIN PROGRAM, data input. This number indicates that the first part of the data tape has been read and that the computer is ready for the second part of the data.
3	This number and those which follow are all used in the COMPUTATION OF U AND V COORDINATES. Each pause indicates that a certain quantity is equal to zero. The natural logarithm of this quantity is required by the program. Since these quantities are equal to zero, the logarithms are undefined, and the complex potential of flow cannot be evaluated. This pause number indicates that 1.0 + s = 0.
4	Indicates that 1.0 - s = 0
5	Indicates that h + s = 0
6	Indicates that h - s = 0

Derivation of Pressure Coefficient Discrepancy

For the case of no loss the pressure coefficient is defined as:

$$C_p = \frac{P - P_o}{\rho V_o^2} = 1 - \left(\frac{V}{V_o}\right)^2 \quad \text{(See the derivation in the report)}$$

With losses, Bernoulli's equation becomes

$$\rho K \frac{V_a^2}{2} + \rho \frac{V_a^2}{2} + P_a = \rho \frac{V_b^2}{2} + P_b$$

This expression is for two points of equal elevation.

Let "a" be a point downstream from the offset, and "b" represent a point upstream from the offset in the region of uniform flow. Then,

$$P - P_o = \rho \frac{V_o^2}{2} - \rho \frac{V^2}{2} - \rho K \frac{V^2}{2}$$

and

$$(C_p)_{actual} = \frac{P - P_o}{\rho \frac{V_o^2}{2}} = 1 - (1 + K)\left(\frac{V}{V_o}\right)^2 \tag{8}$$

The difference between the two values of the pressure coefficient is equal to the pressure coefficient discrepancy, ΔC_p.

Therefore,

$$\Delta C_p = (C_p)_{actual} - C_p = 1 - (1 + K)\left(\frac{V}{V_o}\right)^2 - 1 + \left(\frac{V}{V_o}\right)^2$$

and

$$\Delta C_p = K\left(\frac{V}{V_o}\right)^2 \tag{9}$$

From the equation of continuity,

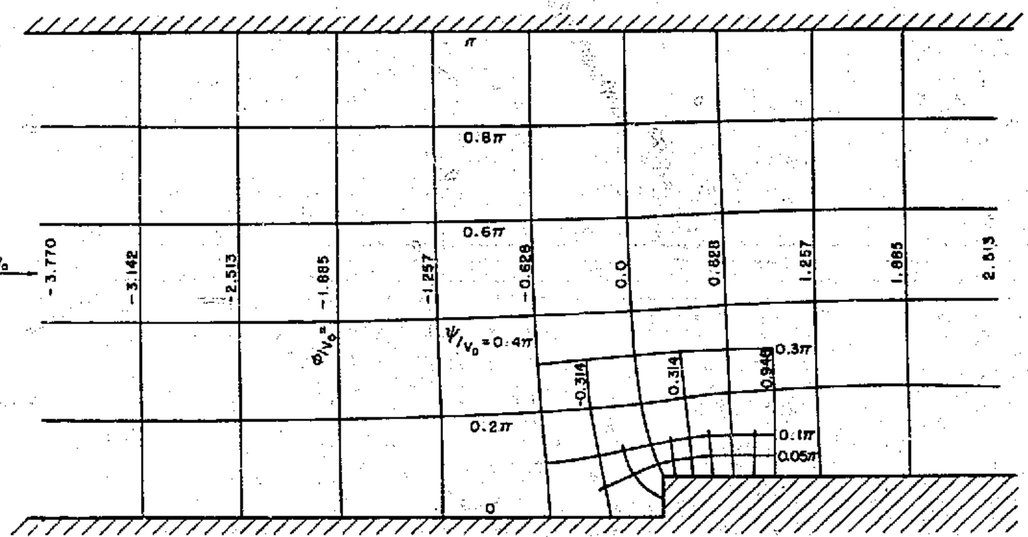
$$Q = V_o\ 1 = V \cdot h$$

or

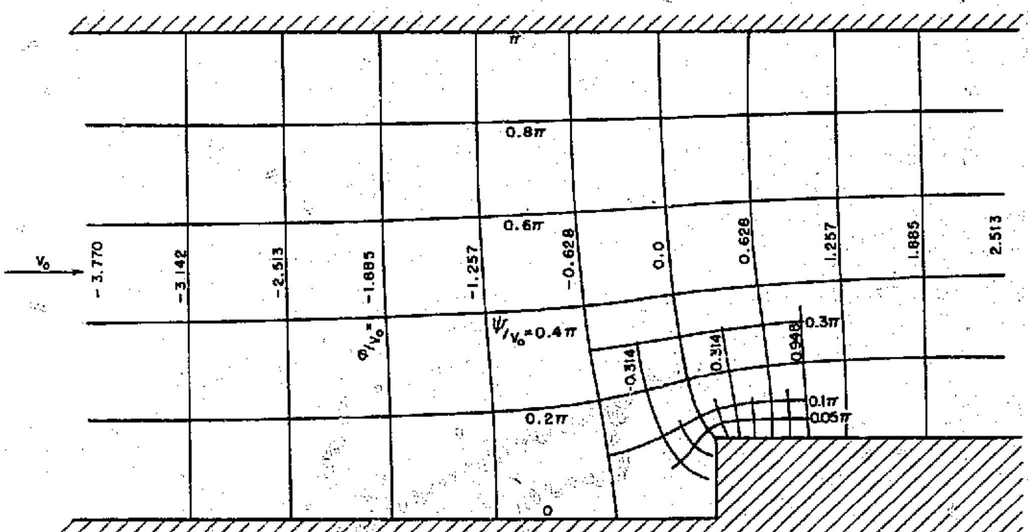
$$V = \frac{V_o}{h}$$

Substituting this value of V into Equation (9) gives the desired relationship,

$$\Delta C_p = K\left(\frac{1}{h}\right)^2 = \frac{K}{h^2} \tag{10}$$



h = 0.917

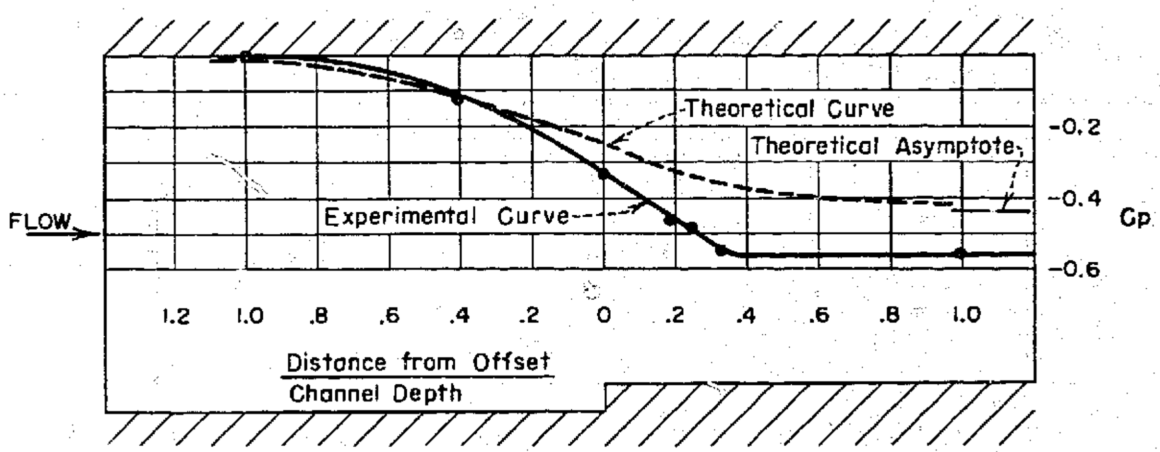


h = 0.833

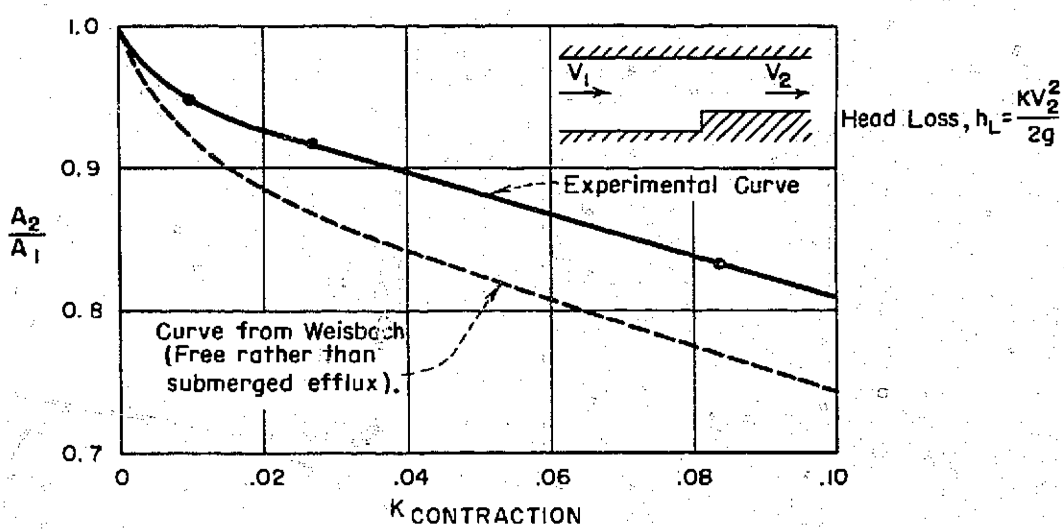
$$h = \frac{\text{Depth of Downstream Channel}}{\text{Depth of Upstream Channel}}$$

COMPUTER SOLUTION OF FLOW NETS FOR 90° BOUNDARY OFFSETS

FLOW NETS FOR OFFSETS GIVING DEPTH RATIOS OF 0.917 AND 0.833

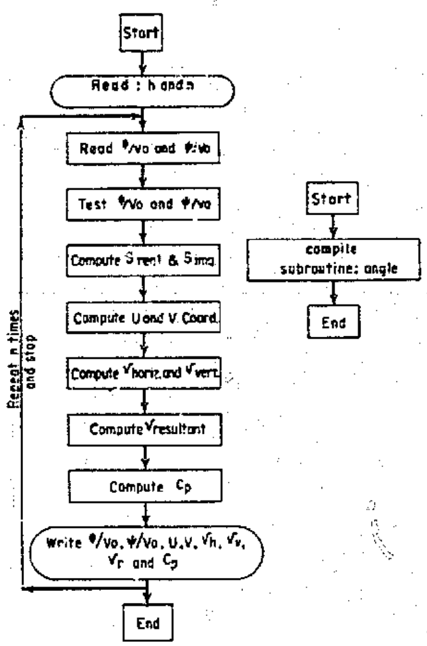


A. PRESSURE COEFFICIENT ALONG TOP OF CONDUIT
$h = 0.833$

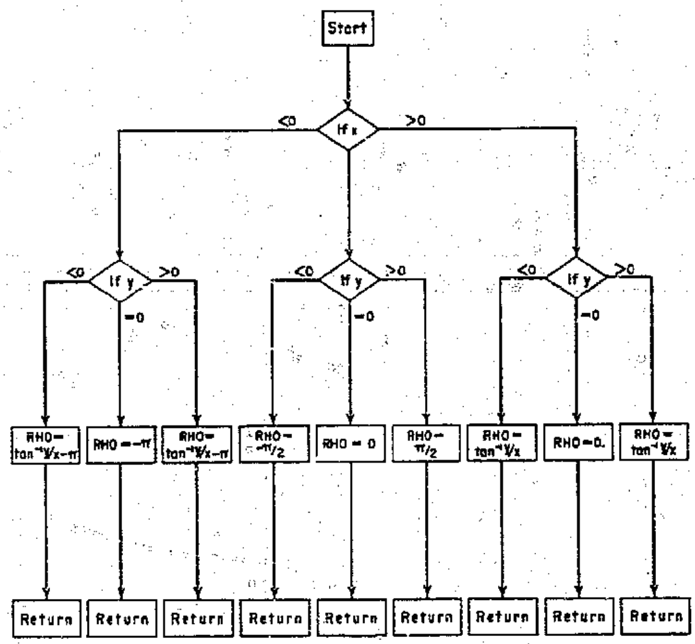


B. LOSS COEFFICIENT FOR SUDDEN CONTRACTION

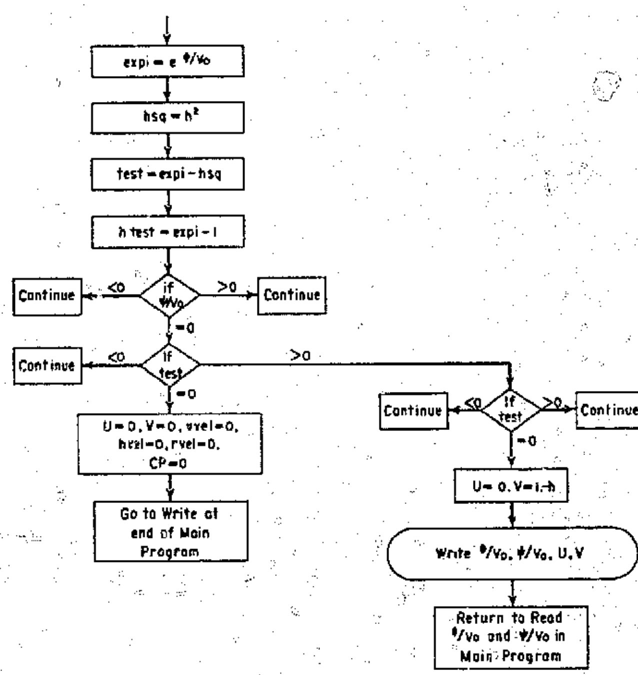
COMPUTER SOLUTION OF FLOW NETS FOR 90° BOUNDARY OFFSETS
PRESSURE AND LOSS COEFFICIENTS FOR OFFSETS



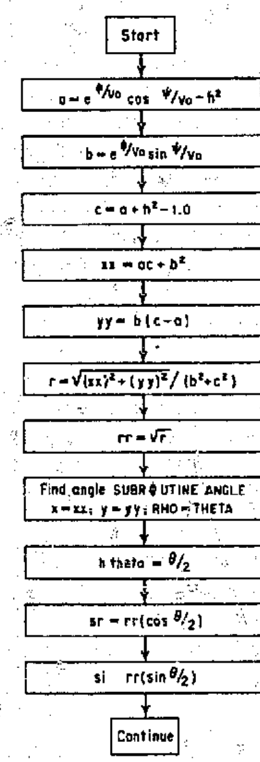
A. MAIN PROGRAM



B. SUBRØUTINE ANGLE



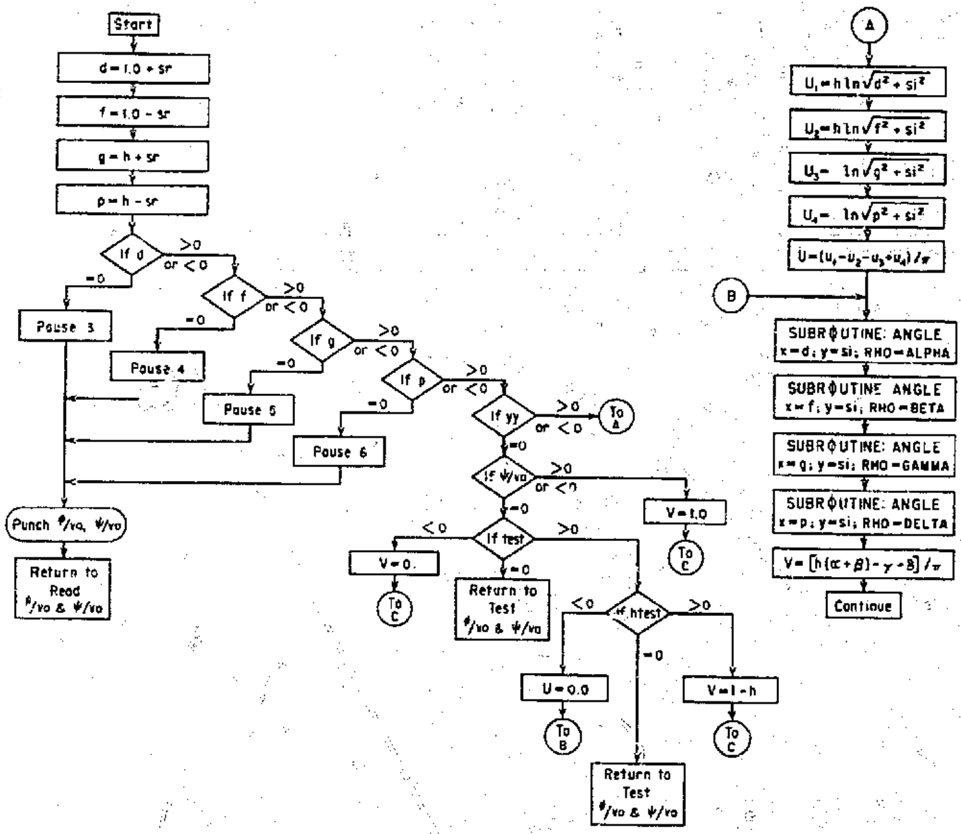
C. TEST PHI AND PSI



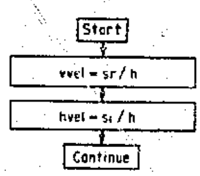
D. COMPUTATION OF S_{REAL} AND $S_{IMAGINARY}$

COMPUTER SOLUTION OF FLOW NETS FOR 90° BOUNDARY OFFSETS

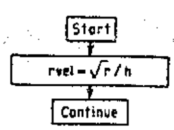
BLOCK DIAGRAMS



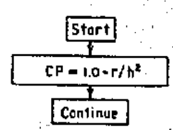
A. COMPUTATION OF U AND V COORDINATES



B. COMPUTATION OF VERTICAL AND HORIZONTAL VELOCITY COMPONENTS



C. COMPUTATION OF RESULTANT VELOCITY BLOCK DIAGRAM



D. COMPUTATION OF PRESSURE COEFFICIENT BLOCK DIAGRAM

COMPUTER SOLUTION OF FLOW NETS FOR 90° BOUNDARY OFFSETS

BLOCK DIAGRAMS

GPO 838-296